**Chapter 6 –Circuit Equations**

**P6.1.3** (a) Determine *VO* in Figure P6.1.3 and the voltages of the middle nodes, taking the lower node as reference. (b) Repeat (a) taking node ‘n’ as reference.



**Solution:** (a) The node-voltage equations are:

Node ‘a’: (1/2 + 1/6)*Va* – 1/2*Vb* – 1/6*Vc* = 4

or, 4*Va* – 3*Vb* – *Vc* = 24

Node ‘b’: -1/2*Va* + (1/2 + 1/2 + 1/4)*Vb* – 1/4*Vc* = 0

or, -2*Va* + 5*Vb* – *Vc* = 0

Node ‘c’: -1/6*Va* – 1/4Vb + (1/4 + 1/6 + 1/6)*Vc* = 0

or, -2*Va* – 3*Vb* + 7*Vc* = 0

Solving these equations gives *Va* = 12 V, *Vb* = 6 V, *Vc* = 6 V.



**P6.1.6** Given that *Va* = 25 V and *Vb* = 12 V, with node ‘c’ grounded. Determine *Va* if node ‘b’ is grounded instead of node ‘c’.

(b) To obtain the voltages with respect to node ‘n’ as reference, 6 V is subtracted from the node voltages in (a): voltage of upper node is 6 V, voltage of middle nodes is zero and voltage of lower node is – 6 V.

**Solution:** The voltage of node ‘b’ is subtracted from that of node ‘a’ to give 13 V.

**P6.1.13** Determine *VO* in Figure P6.1.13.



**Solution:** Considering that a current source  is connected to node ‘a’, the node voltage equation for this node may be written as: =  . This may be rearranged as: 1.25*Va* – 0.5*Vb* = -7.5. The equation for node ‘b’ is: -0.5*Va* + 0.625*Vb* = 10 – 4*Ix* = 10 – 2*Va*, which may be rearranged as: . Solving, *Va* = 0.204 V and *Vb* = 15.51 V = *VO*.

**P6.1.22** Determine *VO* in Figure P6.1.22, assuming that all resistances are 2 Ω.



**Solution:** The resistance in series with the 5 A source is redundant as far as *VO* is concerned, and the CCVS is equivalent to a 0.5 Ω resistance. Making these changes, the circuit becomes as shown. The node voltage equations are:

Node ‘a’: 0.4*Va* – 0.4*Vd* = 5 – *Ix*

Node ‘c’: -0.5*Vb* + 1.5*Vc* – 0.5*Vd* – 0.5*Ve* = *Ix*. adding these two equations:

0.4*Va* – 0.5*Vb* + 1.5*Vc* – 0.9*Vd* – 0.5*Ve* = 5, with *Va* – *Vc* = 10.

Node ‘b’: *Vb* – 0.5*Vc* = -5

Node ‘d’: -0.4*Va* – 0.5*Vc* + 0.9*Vd* = 0.5*Ix*, where *Ix* = 5 – 0.4*Va* + 0.4*Vd*, or

-0.2*Va* – 0.5*Vc* + 0.7*Vd* = 2.5

Node ‘e’: -0.5*Vc* + 1.5*Ve* = 0. Solving these equations gives: *Vc* = 2.338 V and *Ve* = 0.779 V, so that *VO* = 2.338 – 0.779 = 1.56 V.

**P6.2.4** Determine *VO* in Figure P6.2.4.



**Solution:** The mesh-current equations are:

80*I*1 – 20*I*2 – 40*I*3 = 80

-20*I*1 + 160*I*2 – 100*I*3 = 0

-40*I*1 – 100*I*2 + 160*I*3 = 0

The solution is: *I*1 = 1.625 A, *I*2 = 0.750 A, and *I*3 = 0.875 A, which gives *Vbc* = *VO* = 12.5 V.

**P6.2.11** Determine *IO* in Figure P6.2.11.



**Solution:** Transforming the current source to its equivalent voltage source and changing conductances to resistances, the circuit becomes as shown. The mesh-current equations are:



Mesh 1: 1.25*I*1 – 0.5*I*2 = -7.5

Mesh 2: -0.5*I*1 + 0.625*I*2 = 10 – 4*Vx*, where *Vx* = 0.5*I*1, or 1.5*I*1 + 0.625*I*2 = 10. Solving these equations, gives, *I*2 = *IO* = 15.51 A.

**P6.2.14** Determine *IO* in Figure P6.2.12.



**Solution:** Changing the conductances to resistances, the circuit becomes as shown. The mesh-current equations are:



Mesh 1: *I*1 – 0.5*I*2 – 0.5*I*3 = 10

Mesh 2: *I*2 = -2A

Mesh 3: -0.5*I*1 + 0.75*I*3 = -*Vx*

Mesh 4: -0.5*I*2 + 0.75*I*4 = +*Vx*

Adding these equations: -0.5*I*1 – 0.5*I*2 + 0.75*I*3 + 0.75*I*4 = 0.

For the dependent current source, 2*Ix* = *I*4 – *I*3, where *Ix* = *I*2 – *I*1, or

2*I*1 – 2*I*2 – *I*3 + *I*4 = 0. Solving these equations gives *I*4 = *IO* = -22 A.

**P6.2.17** Determine the power delivered or absorbed by each independent source.



**P6.2.17** Mesh 1: *I*1 = 10 A.



Mesh 2: -2*I*1 + 6*I*2 – 2*I*4 = -2*Vx*, where

*Vx* = -2*I*2, or –*I*1 + *I*2 – *I*4 = 0

Mesh 3: -4*I*1 + 8*I*3 – 4*I*4 = 2*Vx*; or

-*I1*+ *I*2 + 2*I*3 – *I*4 = 10.

Mesh 4: -2*I*2 – 4*I*3 + 6*I*4 = -

20, or -*I*2 – 2*I*3 + 3*I*4 = -10.

Solving, *I*2 = 10 A, *I*3 = 0, and *I*4 =

0. The voltage across the 10 A source is 4*I*1 + 2(*!*1 – *I*2) + 4(*I*1 – *I*3) =

80 V. The 10 A source therefore delivers 800 W. The current through the 20 V source is zero, so this source neither absorbs nor delivers power.